Supplemental WS #6

① RHS $= \csc\theta - \sin\theta$

$= \dfrac{1}{\sin\theta} - \sin\theta$

$= \dfrac{1}{\sin\theta} - \dfrac{\sin^2\theta}{\sin\theta}$

$= \dfrac{1-\sin^2\theta}{\sin\theta}$

$= \dfrac{\cos^2\theta}{\sin\theta}$

$= \cos\theta \cdot \dfrac{\cos\theta}{\sin\theta}$

$= \cos\theta \cdot \cot\theta$

$=$ LHS

② LHS $= \dfrac{\sin^2\theta}{1+\cos\theta}$

$= \dfrac{1-\cos^2\theta}{1+\cos\theta}$

$= \dfrac{(1+\cos\theta)(1-\cos\theta)}{1+\cos\theta}$

$= 1-\cos\theta$

$=$ RHS

③ LHS $= \dfrac{1}{1+\sin\theta} + \dfrac{1}{1-\sin\theta}$

$= \dfrac{1(1-\sin\theta)}{1+\sin\theta(1-\sin\theta)} + \dfrac{1(1+\sin\theta)}{1-\sin\theta(1+\sin\theta)}$

$= \dfrac{1-\sin\theta+1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$

$= \dfrac{2}{1-\sin^2\theta}$

$= \dfrac{2}{\cos^2\theta}$

$= 2\sec^2\theta$

$=$ RHS

④ LHS $= \tan^2\theta - \sin^2\theta$

$= \dfrac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$

$= \sin^2\theta\left(\dfrac{1}{\cos^2\theta} - 1\right)$

$= \sin^2\theta\,(\sec^2\theta - 1)$ ← $\tan^2\theta + 1 = \sec^2\theta$

$= \sin^2\theta \cdot \tan^2\theta$

$=$ RHS

⑤ RHS $= \dfrac{1}{\tan\theta + \cot\theta}$

$= \dfrac{1}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}}$

$= \dfrac{1}{\frac{\sin\theta}{\cos\theta}\cdot\frac{\sin\theta}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\cdot\frac{\cos\theta}{\cos\theta}}$

$= \dfrac{1}{\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cdot\cos\theta}}$

$= \dfrac{\sin\theta\cdot\cos\theta}{\sin^2\theta + \cos^2\theta}$

$= \sin\theta\cdot\cos\theta$

$= \dfrac{\sin\theta}{\sec\theta}$

$=$ LHS

⑥ LHS $= \dfrac{1}{\sec\theta} + \dfrac{\sin\theta}{\cot\theta}$

$= \cos\theta + \dfrac{\sin\theta}{\frac{\cos\theta}{\sin\theta}}$

$= \cos\theta + \sin\theta \cdot \dfrac{\sin\theta}{\cos\theta}$

$= \cos\theta \left(\dfrac{\cos\theta}{\cos\theta}\right) + \dfrac{\sin^2\theta}{\cos\theta}$

$= \dfrac{\cos^2\theta + \sin^2\theta}{\cos\theta}$

$= \dfrac{1}{\cos\theta}$

$= \sec\theta$

$=$ RHS

⑦ LHS $= \dfrac{1 - \cos^2\theta}{1 - \sin^2\theta}$

$= \dfrac{\sin^2\theta}{\cos^2\theta}$

$= \dfrac{\sin\theta}{\cos\theta} \cdot \dfrac{\sin\theta}{\cos\theta}$

$= \tan\theta \cdot \tan\theta$

$= \tan\theta \cdot \dfrac{1}{\cot\theta}$

$= \dfrac{\tan\theta}{\cot\theta}$

$=$ RHS

(8) LHS $= \sin^4\theta - \cos^4\theta$

$= (\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)$

$= (1)(\sin^2\theta - \cos^2\theta)$

$= \sin^2\theta - \cos^2\theta$

$=$ RHS

(9) note: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

LHS $= \dfrac{\sin^3\theta - \cos^3\theta}{\sin\theta - \cos\theta}$

$= \dfrac{(\sin\theta - \cos\theta)(\sin^2\theta + \sin\theta\cos\theta + \cos^2\theta)}{(\sin\theta - \cos\theta)}$

$= \sin^2\theta + \sin\theta \cdot \cos\theta + \cos^2\theta$

$= 1 + \sin\theta \cdot \cos\theta$

$=$ RHS

(ii) LHS $= \tan\theta(\sin^2\theta) - \cot\theta(\cos^2\theta)$

$= \frac{\sin\theta}{\cos\theta}(\sin^2\theta) - \frac{\cos\theta}{\sin\theta}(\cos^2\theta)$

$= \frac{\sin^3\theta}{\cos\theta} - \frac{\cos^3\theta}{\sin\theta}$

$= \frac{\sin^3\theta}{\cos\theta}\cdot\frac{\sin\theta}{\sin\theta} - \frac{\cos^3\theta}{\sin\theta}\cdot\frac{\cos\theta}{\cos\theta}$

$= \frac{\sin^4\theta - \cos^4\theta}{\sin\theta\cdot\cos\theta}$

$= \frac{(\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)}{\sin\theta\cdot\cos\theta}$

$= \frac{\sin^2\theta - \cos^2\theta}{\sin\theta\cdot\cos\theta}$

$= \frac{\sin^2\theta}{\sin\theta\cdot\cos\theta} - \frac{\cos^2\theta}{\sin\theta\cdot\cos\theta}$

$= \frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}$

$= \tan\theta - \cot\theta$

$=$ RHS

(11) LHS $= \dfrac{\cos\theta}{1+\sin\theta} + \dfrac{1+\sin\theta}{\cos\theta}$

$= \dfrac{\cos\theta}{1+\sin\theta}\left(\dfrac{\cos\theta}{\cos\theta}\right) + \dfrac{1+\sin\theta}{\cos\theta}\left(\dfrac{1+\sin\theta}{1+\sin\theta}\right)$

$= \dfrac{\cos^2\theta + 1 + 2\sin\theta + \sin^2\theta}{(1+\sin\theta)\cos\theta}$

$= \dfrac{1+1+2\sin\theta}{(1+\sin\theta)\cos\theta}$

$= \dfrac{2+2\sin\theta}{(1+\sin\theta)\cos\theta}$

$= \dfrac{2(1+\sin\theta)}{(1+\sin\theta)\cos\theta}$

$= \dfrac{2}{\cos\theta}$

$= 2\sec\theta$

$=$ RHS

(12) LHS $= \tan\theta + \dfrac{\cos\theta}{1+\sin\theta}$

$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{1+\sin\theta}$

$= \dfrac{\sin\theta}{\cos\theta}\left(\dfrac{1+\sin\theta}{1+\sin\theta}\right) + \dfrac{\cos\theta}{1+\sin\theta}\left(\dfrac{\cos\theta}{\cos\theta}\right)$

$= \dfrac{\sin\theta + \sin^2\theta + \cos^2\theta}{\cos\theta\,(1+\sin\theta)}$

$= \dfrac{\sin\theta + 1}{\cos\theta\,(1+\sin\theta)}$

$= \dfrac{1}{\cos\theta}$

$= \sec\theta$

$=$ RHS

(13) LHS $= \dfrac{\sec^2\theta}{1+\csc^2\theta} - \dfrac{1}{\cos^2\theta} + 1$

$= \dfrac{\sec^2\theta}{\csc^2\theta} - \dfrac{1}{\cos^2\theta} + 1$

$= \dfrac{\frac{1}{\cos^2\theta}}{\frac{1}{\sin^2\theta}} - \dfrac{1}{\cos^2\theta} + 1$

$= \dfrac{1}{\cos^2\theta} \cdot \dfrac{\sin^2\theta}{1} - \dfrac{1}{\cos^2\theta} + 1$

$= \dfrac{\sin^2\theta}{\cos^2\theta} - \dfrac{1}{\cos^2\theta} + 1$

$= \dfrac{\sin^2\theta - 1}{\cos^2\theta} + 1$

$= \dfrac{-\cos^2\theta}{\cos^2\theta} + 1$

$= -1 + 1$

$= 0$

$=$ RHS

(14) LHS $= \dfrac{\sec\theta - \csc\theta}{\sec\theta + \csc\theta}$

$= \dfrac{\frac{1}{\cos\theta} - \frac{1}{\sin\theta}}{\frac{1}{\cos\theta} + \frac{1}{\sin\theta}}$

$= \dfrac{\frac{\sin\theta - \cos\theta}{\sin\theta \cdot \cos\theta}}{\frac{\sin\theta + \cos\theta}{\sin\theta \cdot \cos\theta}}$

$= \dfrac{\sin\theta - \cos\theta}{\sin\theta \cdot \cos\theta} \cdot \dfrac{\sin\theta \cdot \cos\theta}{\sin\theta + \cos\theta}$

$= \dfrac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta}$

RHS $= \dfrac{\tan\theta - 1}{\tan\theta + 1}$

$= \dfrac{\frac{\sin\theta}{\cos\theta} - 1}{\frac{\sin\theta}{\cos\theta} + 1}$

$= \dfrac{\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta}}$

$= \dfrac{\frac{\sin\theta - \cos\theta}{\cos\theta}}{\frac{\sin\theta + \cos\theta}{\cos\theta}}$

$= \dfrac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta}$

LHS = RHS