

Supplemental WS # 6

$$\begin{aligned}
 \textcircled{1} \text{ RHS} &= \csc \theta - \sin \theta \\
 &= \frac{1}{\sin \theta} - \sin \theta \\
 &= \frac{1}{\sin \theta} - \frac{\sin^2 \theta}{\sin \theta} \\
 &= \frac{1 - \sin^2 \theta}{\sin \theta} \\
 &= \frac{\cos^2 \theta}{\sin \theta} \\
 &= \cos \theta \cdot \frac{\cos \theta}{\sin \theta} \\
 &= \cos \theta \cdot \cot \theta \\
 &= \text{LHS}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{2} \text{ LHS} &= \frac{\sin^2 \theta}{1 + \cos \theta} \\
 &= \frac{1 - \cos^2 \theta}{1 + \cos \theta} \\
 &= \frac{(1 + \cos \theta)(1 - \cos \theta)}{1 + \cos \theta} \\
 &= 1 - \cos \theta \\
 &= \text{RHS}
 \end{aligned}$$

3

$$\begin{aligned} \text{LHS} &= \frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} \\ &= \frac{1(1-\sin\theta)}{1+\sin\theta(1-\sin\theta)} + \frac{1(1+\sin\theta)}{1-\sin\theta(1+\sin\theta)} \\ &= \frac{1-\sin\theta + 1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)} \\ &= \frac{2}{1-\sin^2\theta} \\ &= \frac{2}{\cos^2\theta} \\ &= 2\sec^2\theta \\ &= \text{RHS} \end{aligned}$$

4

$$\begin{aligned} \text{LHS} &= \tan^2\theta - \sin^2\theta \\ &= \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta \\ &= \sin^2\theta \left(\frac{1}{\cos^2\theta} - 1 \right) \\ &= \sin^2\theta (\sec^2\theta - 1) \quad \leftarrow \tan^2\theta + 1 = \sec^2\theta \\ &= \sin^2\theta \cdot \tan^2\theta \\ &= \text{RHS} \end{aligned}$$

$$\textcircled{5} \text{ RHS} =$$

$$\frac{1}{\tan \theta + \cot \theta}$$

$$= \frac{1}{\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}}$$

$$= \frac{1}{\frac{\sin \theta}{\cos \theta} \cdot \frac{\sin \theta}{\sin \theta} + \frac{\cos \theta}{\sin \theta} \cdot \frac{\cos \theta}{\cos \theta}}$$

$$= \frac{1}{\frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cdot \cos \theta}}$$

$$= \frac{\sin \theta \cdot \cos \theta}{\sin^2 \theta + \cos^2 \theta}$$

$$= \frac{\sin \theta \cdot \cos \theta}{1}$$

$$= \frac{\sin \theta}{\sec \theta}$$

$$= \text{LHS}$$

$$\begin{aligned}
 \textcircled{6} \quad \text{LHS} &= \frac{1}{\sec \theta} + \frac{\sin \theta}{\cot \theta} \\
 &= \cos \theta + \frac{\sin \theta}{\frac{\cos \theta}{\sin \theta}} \\
 &= \cos \theta + \sin \theta \cdot \frac{\sin \theta}{\cos \theta} \\
 &= \cos \theta \left(\frac{\cos \theta}{\cos \theta} \right) + \frac{\sin^2 \theta}{\cos \theta} \\
 &= \frac{\cos^2 \theta + \sin^2 \theta}{\cos \theta} \\
 &= \frac{1}{\cos \theta} \\
 &= \sec \theta \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{7} \quad \text{LHS} &= \frac{1 - \cos^2 \theta}{1 - \sin^2 \theta} \\
 &= \frac{\sin^2 \theta}{\cos^2 \theta} \\
 &= \frac{\sin \theta}{\cos \theta} \cdot \frac{\sin \theta}{\cos \theta} \\
 &= \tan \theta \cdot \tan \theta \\
 &= \tan \theta \cdot \frac{1}{\cot \theta} \\
 &= \frac{\tan \theta}{\cot \theta} \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{8} \quad \text{LHS} &= \sin^4 \theta - \cos^4 \theta \\
 &= (\sin^2 \theta + \cos^2 \theta) (\sin^2 \theta - \cos^2 \theta) \\
 &= (1) (\sin^2 \theta - \cos^2 \theta) \\
 &= \sin^2 \theta - \cos^2 \theta \\
 &= \text{RHS}
 \end{aligned}$$

$$\textcircled{9} \quad \text{note: } a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$\begin{aligned}
 \text{LHS} &= \frac{\sin^3 \theta - \cos^3 \theta}{\sin \theta - \cos \theta} \\
 &= \frac{(\sin \theta - \cos \theta)(\sin^2 \theta + \sin \theta \cos \theta + \cos^2 \theta)}{(\sin \theta - \cos \theta)} \\
 &= \sin^2 \theta + \sin \theta \cdot \cos \theta + \cos^2 \theta \\
 &= 1 + \sin \theta \cdot \cos \theta \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
(10) \text{ LHS} &= \tan \theta (\sin^2 \theta) - \cot \theta (\cos^2 \theta) \\
&= \frac{\sin \theta}{\cos \theta} (\sin^2 \theta) - \frac{\cos \theta}{\sin \theta} (\cos^2 \theta) \\
&= \frac{\sin^3 \theta}{\cos \theta} - \frac{\cos^3 \theta}{\sin \theta} \\
&= \frac{\sin^3 \theta}{\cos \theta} \cdot \frac{\sin \theta}{\sin \theta} - \frac{\cos^3 \theta}{\sin \theta} \cdot \frac{\cos \theta}{\cos \theta} \\
&= \frac{\sin^4 \theta}{\sin \theta \cdot \cos \theta} - \frac{\cos^4 \theta}{\sin \theta \cdot \cos \theta} \\
&= \frac{(\sin^4 \theta + \cos^4 \theta) (\sin^2 \theta - \cos^2 \theta)}{\sin \theta \cdot \cos \theta} \\
&= \frac{\sin^2 \theta - \cos^2 \theta}{\sin \theta \cdot \cos \theta} \\
&= \frac{\sin^2 \theta}{\sin \theta \cdot \cos \theta} - \frac{\cos^2 \theta}{\sin \theta \cdot \cos \theta} \\
&= \frac{\sin \theta}{\cos \theta} - \frac{\cos \theta}{\sin \theta} \\
&= \tan \theta - \cot \theta \\
&= \text{RHS}
\end{aligned}$$

$$\begin{aligned}
\textcircled{11} \quad \text{LHS} &= \frac{\cos \theta}{1 + \sin \theta} + \frac{1 + \sin \theta}{\cos \theta} \\
&= \frac{\cos \theta}{1 + \sin \theta} \left(\frac{\cos \theta}{\cos \theta} \right) + \frac{1 + \sin \theta}{\cos \theta} \left(\frac{1 + \sin \theta}{1 + \sin \theta} \right) \\
&= \frac{\cos^2 \theta + 1 + 2 \sin \theta + \sin^2 \theta}{(1 + \sin \theta) \cos \theta} \\
&= \frac{1 + 1 + 2 \sin \theta}{(1 + \sin \theta) \cos \theta} \\
&= \frac{2 + 2 \sin \theta}{(1 + \sin \theta) \cos \theta} \\
&= \frac{2(1 + \sin \theta)}{(1 + \sin \theta) \cos \theta} \\
&= \frac{2}{\cos \theta} \\
&= 2 \sec \theta \\
&= \text{RHS}
\end{aligned}$$

(12)

$$\text{LHS} = \tan \theta + \frac{\cos \theta}{1 + \sin \theta}$$

$$= \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{1 + \sin \theta}$$

$$= \frac{\sin \theta}{\cos \theta} \left(\frac{1 + \sin \theta}{1 + \sin \theta} \right) + \frac{\cos \theta}{1 + \sin \theta} \left(\frac{\cos \theta}{\cos \theta} \right)$$

$$= \frac{\sin \theta + \sin^2 \theta + \cos^2 \theta}{\cos \theta (1 + \sin \theta)}$$

$$= \frac{\sin \theta + 1}{\cos \theta (1 + \sin \theta)}$$

$$= \frac{1}{\cos \theta}$$

$$= \sec \theta$$

$$= \text{RHS}$$

13

LHS =

$$\frac{\sec^2 \theta}{1 + \cot^2 \theta} - \frac{1}{\cos^2 \theta} + 1$$

$$= \frac{\sec^2 \theta}{\csc^2 \theta} - \frac{1}{\cos^2 \theta} + 1$$

$$= \frac{1}{\cos^2 \theta} \cdot \frac{1}{\sin^2 \theta} - \frac{1}{\cos^2 \theta} + 1$$

$$= \frac{1}{\cos^2 \theta} \cdot \frac{\sin^2 \theta}{1} - \frac{1}{\cos^2 \theta} + 1$$

$$= \frac{\sin^2 \theta}{\cos^2 \theta} - \frac{1}{\cos^2 \theta} + 1$$

$$= \frac{\sin^2 \theta - 1}{\cos^2 \theta} + 1$$

$$= \frac{-\cos^2 \theta}{\cos^2 \theta} + 1$$

$$= -1 + 1$$

$$= 0$$

RHS

14

$$\text{LHS} = \frac{\sec \theta - \csc \theta}{\sec \theta + \csc \theta}$$

$$= \frac{1}{\cos \theta} - \frac{1}{\sin \theta}$$

$$\frac{1}{\cos \theta} + \frac{1}{\sin \theta}$$

$$= \frac{\sin \theta - \cos \theta}{\sin \theta \cdot \cos \theta} \cdot \frac{\sin \theta + \cos \theta}{\sin \theta + \cos \theta}$$

$$= \frac{\sin \theta - \cos \theta}{\cancel{\sin \theta \cos \theta}} \cdot \frac{\cancel{\sin \theta \cos \theta}}{\sin \theta + \cos \theta}$$

$$= \frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta}$$

$$\text{RHS} = \frac{\tan \theta - 1}{\tan \theta + 1}$$

$$= \frac{\frac{\sin \theta}{\cos \theta} - 1}{\frac{\sin \theta}{\cos \theta} + 1}$$

$$\frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta}$$

$$= \frac{\frac{\sin \theta}{\cos \theta} - \frac{\cos \theta}{\cos \theta}}{\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\cos \theta}}$$

$$\frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta}$$

$$= \frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta}$$

$$= \frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta}$$

LHS = RHS